$\mathcal{N} = 4$ supergravity amplitudes

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Workshop on Black Holes in Supergravity and M/Superstring Theory
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based on
[arXiv:1202.3692] and [arXiv:1208.1255]
with Piotr Tourkine

Motivations

There are intriguing relations between supergravity and super-Yang-Mills amplitudes:

- Color-kinematic duality [see talk by Bern at Nicolai-fest]
- structure in the $1/N$-expansion

These give some support to the relation that SUGRA field theory amplitudes can be presented as the square of SYM amplitudes

$$SUGRA \sim (SYM)^2$$

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At tree-level the relation $SUGRA \sim (SYM)^2$ can be understood simply by the use of the momentum kernel formalism [Kawai, Lewellen, Tye] [Bjerrum-Bohr, Damgaard, Feng, Sondergaard], [Bjerrum-Bohr, Damgaard, Sondergaard, Vanhove]

Gravity $\mathfrak{M}^{tree}$ and Yang-Mills $\mathfrak{A}^{tree}$ amplitudes are build from the same data

$$\begin{aligned}\mathfrak{A}_n^{tree} &= A_n^{\text{spin }1} \otimes S(k_i \cdot k_j) \otimes A_n^{\text{spin }0}\\ \mathfrak{M}_n^{tree} &= A_n^{\text{spin }1} \otimes S(k_i \cdot k_j) \otimes A_n^{\text{spin }1}\end{aligned}$$

- $A_n^{\text{spin s}}$ are the color-stripped amplitudes for spin $s = 1, 0$
- there are in the kernel of the momentum kernel

$$\sum_{\sigma \in \mathfrak{S}_{n-2}} S[\beta|\sigma]\, A_n^{spin\ s}(1, \sigma(2, \ldots, n-1), n) = 0, \qquad \forall \beta \in \mathfrak{S}_{n-2}$$

- $S(k_i \cdot k_j)$ depends only the kinematics invariants

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$$\begin{aligned} \mathfrak{A}_n^{tree} &= A_n^{\text{spin } 1} \otimes S(k_i \cdot k_j) \otimes A_n^{\text{spin } 0} \\ \mathfrak{M}_n^{tree} &= A_n^{\text{spin } 1} \otimes S(k_i \cdot k_j) \otimes A_n^{\text{spin } 1} \end{aligned}$$

This implies the [Bern-Carrasco-Johansson] parametrization of the amplitudes

$$\mathfrak{A}_n^{\text{tree}} = \sum_{\gamma \in \Gamma_n^{\text{tree}}} \frac{n_\gamma c^\gamma}{\prod_{\alpha \in \gamma} p_\alpha^2}; \qquad \mathfrak{M}_n^{\text{tree}} = \sum_{\gamma \in \Gamma_n^{\text{tree}}} \frac{n_\gamma \tilde{n}^\gamma}{\prod_{\alpha \in \gamma} p_\alpha^2}$$

- Γ_n: φ^3 cubic graphs
- lorentz factor n_γ and color factors c^γ

$SUGRA \sim (SYM)^2$

At loop order this squaring relation require some care.

Since $1 \otimes 1 = 2 \oplus 0$, in the bosonic case propagate extra scalars

In supergravity theories the scalars parametrize a moduli space G/H and the invariant under the global symmetry G put strong constraints on the amplitudes

[Green, Russo, Vanhove], [Elvang, Kiermaier, Freedman et al.], [Bossard, Stelle, Howe]

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The $\mathcal{N} = 8$ supergravity multiplet is the tensor product of two $\mathcal{N} = 4$ SYM gauge multiplets

$$(2|_{\mathbf{1}}, \frac{3}{2}|_{\mathbf{8}}, 1|_{\mathbf{28}}, \frac{1}{2}|_{\mathbf{56}}, 0_{\mathbf{70}})_{\mathcal{N}=8} = (1|_{\mathbf{1}}, \frac{1}{2}|_{\mathbf{4}}, 0_{\mathbf{6}})_{\mathcal{N}=4} \otimes (1|_{\mathbf{1}}, \frac{1}{2}|_{\bar{\mathbf{4}}}, 0|_{\mathbf{6}})_{\mathcal{N}=4}$$

where we have indicated the $SU(8)$ and $SU(4)$ R-symmetry indices

The **70** scalars ϕ^i of the supergravity multiplet parametrize the coset $E_{7(7)}/(SU(8)/\mathbb{Z}_2)$.

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In the case $\mathcal{N} = 4$ SUGRA the gravity multiplet there is no unicity

$$\begin{aligned}
(2|_{\mathbf{1}}, \frac{3}{2}|_{\mathbf{4}}, 1|_{\mathbf{6}}, \frac{1}{2}|_{\mathbf{4}}, 0_{\mathbf{2}})_{\mathcal{N}=4} &= (1|_{\mathbf{1}}, \frac{1}{2}|_{\mathbf{4}}, 0_{\mathbf{6}})_{\mathcal{N}=4} \otimes (1|_{\mathbf{1}})_{\mathcal{N}=0} \\
&= (1_{\mathbf{1}}, \frac{1}{2}|_{\mathbf{2}}, 2 \times 0|_{\mathbf{1}})_{\mathcal{N}=2} \otimes (1_{\mathbf{1}}, \frac{1}{2}|_{\bar{\mathbf{2}}}, 2 \times 0|_{\mathbf{1}})_{\mathcal{N}=2}
\end{aligned}$$

The 2 scalars ϕ^i parametrize the coset $SU(1,1)/U(1)$

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[Green, Russo, Vanhove], [Elvang, Kiermaier, Freedman et al.], [Bossard, Stelle, Howe]

In this talk we will discuss the UV perturbative behaviour of $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supergravity and discuss

- the role of supersymmetry in perturbative computation
- the role of non-perturbative duality symmetries in string theory

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There is no intellectual exercise that is not ultimately pointless
Jorge Luis Borges

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Part I

$\mathcal{N} = 4$ SYM

Constraints from supersymmetry: $\mathcal{N} = 4$ SYM

- The case of $\mathcal{N} = 4$ super-Yang-Mills

$$\mathcal{S} = \frac{1}{g_{\rm YM}^2} \int d^4x \mathrm{tr}(F^2) + \cdots$$

- Coupling constant dimensionless in $D = 4$

Half of supersymmetries are enough for finiteness of $\mathcal{N} = 4$ SYM in $D = 4$

[Mandelstam; Howe, Stelle, West; Brink, Lindgren, Nilsson]

- Four points amplitude behave as

$$\mathfrak{A}_{4;L}^{(D)} \sim \Lambda^{(D-4)L-4}\, t_8 F^4$$

But this is not enough for understanding the correct critical ultraviolet behaviour in the single trace sector, and double trace sector of the 4-point amplitudes in dimensions $4 < D \leqslant 10$

[Berkovits, Green, Russo, Vanhove], [Bern et al.]

Constraints from supersymmetry: $\mathcal{N} = 4$ SYM

$$[\mathfrak{A}^{(D)}_{4;L}] = \Lambda^{(D-4)L-4-2\gamma_L}\, \partial^{2\gamma_L}\, t_8 \mathrm{tr} F^4 + \Lambda^{(D-4)L-4-2\beta_L}\, \partial^{2\beta_L}\, t_8 (\mathrm{tr} F^2)^2$$

	L=1	L=2	L=3	L=4	L=5
$\partial^{2\gamma_L}\, t_8 \mathrm{tr}(F^4)$	$D_c = 8$ $\gamma_1 = 0$	$D_c = 7$ $\gamma_2 = 1$	$D_c = 6$ $\gamma_3 = 1$	$D_c = \frac{11}{2}$ $\gamma_4 = 1$	$D_c = \frac{26}{5}$ $\gamma_5 = 1$
$\partial^{2\beta_L}\, t_8 (\mathrm{tr} F^2)^2$	$D_c = 8$ $\beta_1 = 0$	$D_c = 7$ $\beta_2 = 1$	$D_c = \frac{20}{3}$ $\beta_3 = 2$	$D_c = 6$ $\beta_4 = 2$	$D_c = \frac{28}{5}$ $\beta_5 = 2$

- Some F-term are in $D < 10$

$$\partial^2 t_8 \mathrm{tr}(F^4) \sim \int d^8\theta\, \mathrm{Tr}(W_\alpha^4)$$

$$\partial^4 t_8 (\mathrm{tr}(F^2))^2 \sim \int d^{12}\theta\, (\mathrm{Tr}(W_\alpha^2))^2$$

- Gaugino superfield $D_{(\beta} W_{\alpha)} = (\gamma^{mn})_{\alpha\beta}\, F_{mn}$

Constraints from supersymmetry: $\mathcal{N}=4$ SYM

$$[\mathfrak{A}^{(D)}_{4;L}] = \Lambda^{(D-4)L-4-2\gamma_L}\,\partial^{2\gamma_L}\,t_8\mathrm{tr}F^4 + \Lambda^{(D-4)L-4-2\beta_L}\,\partial^{2\beta_L}\,t_8(\mathrm{tr}F^2)^2$$

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- Some F-term are descendant of the Konishi operator $\mathrm{tr}(\Phi\cdot\Phi)$ in $D<10$

$$\partial^2 t_8\mathrm{tr}(F^4) \sim \int d^{16}\theta\,\mathrm{tr}(\Phi\cdot\Phi)$$

$$\partial^4 t_8(\mathrm{tr}(F^2))^2 \sim \int d^{16}\theta\,(\mathrm{tr}(\Phi\cdot\Phi))^2$$

- These operators are not protected from quantum corrections

Constraints from supersymmetry: $\mathcal{N}=4$ SYM

$$[\mathfrak{A}^{(D)}_{4;L}] = \Lambda^{(D-4)L-4-2\gamma_L}\,\partial^{2\gamma_L}\, t_8 \mathrm{tr} F^4 + \Lambda^{(D-4)L-4-2\beta_L}\,\partial^{2\beta_L}\, t_8 (\mathrm{tr}F^2)^2$$

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For $L \geqslant 4$ the UV divergence is dominated by the single trace term

single trace	$\Lambda^{(D-4)L-6}\,\partial^2 t_8 \mathrm{tr}(F^4)$	$L \geqslant 2$
double trace	$\Lambda^{(D-4)L-8}\,\partial^4 t_8 (\mathrm{tr}F^2)^2$	$L \geqslant 3$

- $\mathcal{N}=3$ superspace explains the leading UV behaviour [Howe, Stelle]
- Confirmed by amplitude computation [Bern, Dixon, Carrasco, Johansson,Roiban] and in $D=5$ by [Bern, Douglas, ...; to appear]

Part II

$\mathcal{N} = 8$ SUGRA

Behavior of supergravity amplitudes

$$\mathcal{S}^{gravity} = \frac{1}{2\kappa^2_{(D)}} \int d^D x \,\sqrt{-g}\,\mathcal{R}$$

- The gravitational coupling constant has dimension $[\kappa^2_{(D)}] = (length)^{D-2}$
- At L-loop gravity amplitudes have the superficial UV behaviour

$$[\mathfrak{M}_L^{(D)}] = \Lambda^{(D-2)L+2}$$

- The 4-graviton amplitudes factorize an $\mathcal{R}^4 = stu\,\mathfrak{M}_4^{\text{tree}}$ term

$$[\mathfrak{M}_L^{(D)}] = \Lambda^{(D-2)L-6-2\beta_L^N}\,\partial^{2\beta_L^N}\mathcal{R}^4$$

- Critical dimension for UV divergences is

$$D \geqslant D_c = 2 + \frac{6+2\beta_L^N}{L}$$

The ultraviolet behaviour of $\mathcal{N}=8$ supergravity

- For $L \leqslant 4$ the UV behaviour is

  [Green,Russo,Vanhove; Bern et al.; Elvang et al.; Bossard et al.]

$$[\mathfrak{M}_{4;L}^{(D)}] \sim \Lambda^{(D-4)L-6}\,\partial^{2L}\,\mathcal{R}^4 \qquad 2 \leqslant L \leqslant 4$$

The same as the one of $\mathcal{N}=4$ SYM

- After 4-loop it is expected that only a $\partial^8\mathcal{R}^4$ is factorized for $L \geqslant 4$

  [Green, Russo, Vanhove; Vanhove; Green, Bjornsson]

$$[\mathfrak{M}_{4;L}^{(D)}] \sim \Lambda^{(D-2)L-14}\,\partial^{8}\,\mathcal{R}^4 \qquad L \geqslant 4$$

worse than the one of $\mathcal{N}=4$ SYM

- At five-loop order the 4-point amplitude in
  - $\mathcal{N}=4$ SYM divergences for $5 < 26/5 \leqslant D$
  - $\mathcal{N}=8$ SUGRA divergences for $24/5 \leqslant D$

Double copy and UV behaviour

The double copy conjecture by BCJ state that the 4 point amplitude in $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ SUGRA are related in a simple way

$$\begin{aligned}
\mathfrak{A}_{4,L}^{(D)} &= g_{\text{YM}}^{2L+2} \sum_{\gamma\in\Gamma_4^L} \int \prod_{i=1}^{L} \frac{d^D\ell_i}{(2\pi)^D} \frac{1}{S_j} \frac{n_\gamma c^\gamma}{\prod_{r=1}^{3L+1} p_r^2} \\
\mathfrak{M}_{4,L}^{(D)} &= \kappa_{(D)}^{2L+2} \sum_{\gamma\in\Gamma_4^L} \int \prod_{i=1}^{L} \frac{d^D\ell_i}{(2\pi)^D} \frac{1}{S_j} \frac{n_\gamma \tilde{n}^\gamma}{\prod_{r=1}^{3L+1} p_r^2} .
\end{aligned}$$

Double copy and UV behaviour

[Vanhove]

For large value of the loop momentum $\ell_i \sim \Lambda \to +\infty$ the amplitude behaves

$$\begin{aligned} \mathfrak{A}_{4,L}^{(D)} &\sim g_{\mathrm{YM}}^{2L+2}\,\Lambda^{DL-2(3L+1)}\,n_\gamma^\infty \\ \mathfrak{M}_{4,L}^{(D)} &\sim \kappa_{(D)}^{2L+2}\,\Lambda^{DL-2(3L+1)}\,(n_\gamma^\infty)^2\,. \end{aligned}$$

The $\mathcal{N}=4$ SYM leading UV behaviour is realized for numerator factor satisfying

- In the simple trace sector

$$n_\gamma^\infty|_{single\ trace} \sim \Lambda^{2L-4} \Longrightarrow \mathfrak{A}_{4,L}^{(D)}|_{single\ trace} \sim \Lambda^{(D-4)L-6}$$

- In the double trace sector

$$n_\gamma^\infty|_{double\ trace} \sim \Lambda^{2L-6} \Longrightarrow \mathfrak{A}_{4,L}^{(D)}|_{double\ trace} \sim \Lambda^{(D-4)L-8}$$

Double copy and UV behaviour

[Vanhove]

For large value of the loop momentum $\ell_i \sim \Lambda \to +\infty$ the amplitude behaves

$$\begin{aligned}\mathfrak{A}^{(D)}_{4,L} &\sim g_{\rm YM}^{2L+2}\,\Lambda^{DL-2(3L+1)}\, n^{\infty}_{\gamma}\\ \mathfrak{M}^{(D)}_{4,L} &\sim \kappa_{(D)}^{2L+2}\,\Lambda^{DL-2(3L+1)}\, (n^{\infty}_{\gamma})^2\,.\end{aligned}$$

This indicates that the leading UV behaviour of $\mathcal{N}=8$ SUGRA is connected to the UV behaviour of SYM in the double trace sector

$$\mathfrak{M}^{(D)}_{4,L} \sim \Lambda^{DL-2(3L+1)}\underbrace{(n^{\infty}_{\gamma}|_{double\ trace})}_{=\Lambda^{2L-6}}{}^2 = \Lambda^{(D-2)L-14}$$

Duality symmetry constraint on UV divergences

The UV behaviour can be derived by an application of the non-renormalisation theorems implied by the action of the extended symmetries and the duality group of $\mathcal{N} = 8$ SUGRA in various dimensions

$$\mathcal{S} = \frac{1}{\ell_D^{D-2}} \int d^D x \, \sqrt{-g} \left(\mathcal{R} + \ell_D^6 \, \mathcal{E}_{(0,0)}^{(D)} \mathcal{R}^4 + \ell_D^{10} \, \mathcal{E}_{(1,0)}^{(D)} D^4 \mathcal{R}^4 + \ell_D^{12} \, \mathcal{E}_{(0,1)}^{(D)} D^6 \mathcal{R}^4 + \cdots \right)$$

The coupling $\mathcal{E}_{(0,0)}^{(D)}$ are function of the scalar fields ϕ^i in the coset $E_{n(n)}(\mathbb{R})/K_n$. In string theory there are automorphic form invariant under the action of $E_{n(n)}(\mathbb{Z})$

A UV divergence is a local counter-term $c_{UV} \, D^{2k} \mathcal{R}^4$ which is a part of the constant term of the string theory automorphic form $\mathcal{E}_{(p,q)}^{(D)}$ with $k = 2p + 3q$

Non-renormalisation theorems

For $D \geqslant 3$ [Green, Russo, Vanhove]

$$\left(\Delta^{(D)} - \frac{3(11-D)(D-8)}{D-2}\right)\mathcal{E}^{(D)}_{(0,0)} = 6\pi\delta_{D-8,0}$$
$$\left(\Delta^{(D)} - \frac{5(12-D)(D-7)}{D-2}\right)\mathcal{E}^{(D)}_{(1,0)} = 80\zeta(2)\delta_{D-7,0}$$
$$\left(\Delta^{(D)} - \frac{6(14-D)(D-6)}{D-2}\right)\mathcal{E}^{(D)}_{(0,1)} = -(\mathcal{E}^{(D)}_{(0,0)})^2 + \zeta(3)\delta_{D-6,0}$$

- The eigenvalues vanish in the critical dimensions for UV divergence $D^{2L}\mathcal{R}^4$ interactions

$$D_c = \begin{cases} 8 & \text{for } L = 1 \\ 4 + 6/L & \text{for } 2 \leqslant L \leqslant 4 \end{cases}$$

- This has been confirmed using various field theory supersymmetry [Bossard, Stelle, Howe], and direct loop computation [Bern, Carrasco, Dixon, Johansson, Roiban], and continuous E_7 arguments [Elvang, Keirmaier, Freedman et al.]

Part III

$\mathcal{N} = 4$ SUGRA

Non-renormalisation theorems

We have seen that supersymmetry and dualities imply various non-renormalisation theorems for higher dimension operators.

- Type II compactifications on a torus ($\mathcal{N} = 8$ models) we had the $\beta_L^8 = L$ rule for $L \leqslant 4$ [Green, Russo, Vanhove; Berkovits]

$$[\mathfrak{M}_{4;L}^{(D)}] = \Lambda^{(D-4)L-6}\, D^{2L}\mathcal{R}^4, \qquad \text{for } L \leqslant 4$$

- Heterotic compactification ($\mathcal{N} = 4$ models)
  - $\mathcal{R}^4$ is a $\frac{1}{2}$-BPS coupling 1-loop exact in perturbation in string theory [Bachas, Kiritsis; Bachas, Fabre, Kiritsis, Obers, Vanhove; Tourkine, Vanhove]

$$\beta_L^4 \geqslant 2 \qquad \text{for } L \geqslant 2$$

Similarly to $\mathcal{N} = 8$ SUGRA these non-renormalisation theorems constraint the UV behaviour of $\mathcal{N} = 4$ supergravity

Constructing $\mathcal{N}=4$ supergravity amplitudes

- The low-energy limit of superstring theory compactified on tori leads to pure $\mathcal{N}=8$ supergravity.
- Obtaining pure $\mathcal{N}=4$ supergravity from string theory is more tricky: generic string models have extra vector multiplets and there is no more a unique way to obtain the $\mathcal{N}=4$ supergraviton multiplet

$$(2|_{\mathbf{1}}, \frac{3}{2}|_{\mathbf{4}}, 1|_{\mathbf{6}}, \frac{1}{2}|_{\mathbf{4}}, 0|_{\mathbf{2}})_{\mathcal{N}=4} = (1|_{\mathbf{1}}, \frac{1}{2}|_{\mathbf{4}}, 0_{\mathbf{6}})_{\mathcal{N}=4} \otimes (1|_{\mathbf{1}})_{\mathcal{N}=0}$$

- or the product of two $\mathcal{N}=2$ SYM gauge multiplets

$$(2|_{\mathbf{1}}, \frac{3}{2}|_{\mathbf{4}}, 1|_{\mathbf{6}}, \frac{1}{2}|_{\mathbf{4}}, 0|_{\mathbf{2}})_{\mathcal{N}=4} = (1_{\mathbf{1}}, \frac{1}{2}|_{\mathbf{2}}, 2\times 0|_{\mathbf{1}})_{\mathcal{N}=2} \otimes (1_{\mathbf{1}}, \frac{1}{2}|_{\bar{\mathbf{2}}}, 2\times 0|_{\mathbf{1}})_{\mathcal{N}=2}$$

Constructing $\mathcal{N} = 4$ supergravity amplitudes

- String theory constructions lead to models that has pure $\mathcal{N} = 4$ SUGRA coupled to $0 \leqslant n_v$ vector multiplet in four dimensions
- The string theory moduli space is (with $\Gamma \subset SL(2, \mathbb{Z})$)

$$\Gamma \backslash SU(1,1,\mathbb{R})/U(1) \times SO(6, n_v, \mathbb{Z}) \backslash SO(6, n_v, \mathbb{R})/SO(6) \times SO(n_v)$$

To get pure $\mathcal{N} = 4$ supergravity we want to set $n_v = 0$ and decouple the string modes.

The string theory effective action is given by

$$\mathcal{S} = \frac{1}{\ell_4^2} \int d^4x\, e \, \left(\mathcal{R} + \ell_4^2 f(S, \bar{S}) R^2 + \ell_4^6\, g(S, \bar{S})\, \mathcal{R}^4 \right)$$

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To get pure $\mathcal{N}=4$ supergravity we want to set $n_v = 0$ and decouple the string modes.

- For $(4,0)$ models the complex scalar S of the supergravity multiplet is in the $SU(1,1,\mathbb{R})/U(1)$ factor
- For $(2,2)$ models the scalar S parametrizes a $SU(1,1,\mathbb{R})/U(1) \subset SO(6,n_v,\mathbb{R})/SO(6) \times SO(n_v)$
- these models are *non-perturbatively dual* in string theory

The predicted UV behaviour for $\mathcal{N} = 4$ supergravity

Taking the limit of one- and two-loop four-graviton amplitudes in string theory [Tourkine, Vanhove] match the supergravity expression for the supergraviton contribution obtained by [Bern et al., Dixon et al., Dunbar et al.]

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We find that the string theory non-renormalisation theorem applies to the pure $\mathcal{N}=4$ supergravity limit and the critical UV behaviour of the 4-graviton amplitude is [Tourkine, Vanhove]

$\mathcal{N}=4$ non-renormalisation theorems for $\mathcal{R}^4$ term $\beta_L^4=1$ for $L\geqslant 2$

$$[\mathfrak{M}_L^{(D)}] = \Lambda^{(D-2)L-8}\,\partial^2\mathcal{R}^4 \qquad \text{for } L\geqslant 2$$

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$$[\mathfrak{M}_L^{(D)}] = \Lambda^{(D-2)L-8}\,\partial^2\mathcal{R}^4 \qquad \text{for } L \geqslant 2$$

- The absence of UV divergence in $D = 4$ at $L = 3$ was obtained from direct field theory computation [Bern, Davies, Dennen, Huang]

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$\mathcal{N} = 4$ non-renormalisation theorems for $\mathcal{R}^4$ term $\beta_L^4 = 1$ for $L \geqslant 2$

$$[\mathfrak{M}_L^{(D)}] = \Lambda^{(D-2)L-8}\, \partial^2 \mathcal{R}^4 \qquad \text{for } L \geqslant 2$$

- The absence of UV divergence in $D = 5$ at $L = 2$ confirmed by direct field theory computation [Bern, Davies, Dennen, Huang (to appear)]

Double copy and UV behaviour

The double copy conjecture procedure for the $(4,0)$ construction instructs us to build the $\mathcal{N}=4$ SUGRA amplitude from the product of numerators from $\mathcal{N}=4$ SYM and pure YM

$$\mathfrak{M}^{(D)}_{4,L} = \kappa^{2L+2}_{(D)} \sum_{\gamma\in\Gamma^L_4} \int \prod_{i=1}^{L} \frac{d^D\ell_i}{(2\pi)^D} \frac{1}{S_j} \frac{n_\gamma|_{\mathcal{N}=4}\tilde n^\gamma|_{YM}}{\prod_{r=1}^{3L+1} p_r^2} .$$

the $\mathcal{N}=4$ supergravity UV behaviour

$$\mathfrak{M}^{(D)}_{4,L} \sim \Lambda^{DL-2(3L+1)} n^\infty_\gamma|_{\mathcal{N}=4 \ double\ trace} \tilde n^\infty_\gamma|_{YM\ subleading} = \Lambda^{(D-2)L-8}$$

is obtained from the subleading UV behaviour in pure YM and $\mathcal{N}=4$ SYM

$$n^\infty_\gamma|_{YM\ leading} \sim \Lambda^{2L+2}; \qquad n^\infty_\gamma|_{YM\ subleading} \sim \Lambda^{2L}$$

Duality symmetries of $\mathcal{N} = 4$ pure supergravity

- In [Bossard, Howe, Stelle, Vanhove] a candidate 3-loop counter-term was constructed using harmonic superspace

$$\int d^4x \int d^{12}\theta (\bar{\chi}\chi)^2 = \int d^4x\, e\, (\mathcal{R}^4 + \text{susy completion})$$

- This term is unique at this order if the duality symmetry $SU(1,1)$ is exact since we showed that the duality invariant supervolume vanishes

$$\int d^4x \int d^{16}\theta E(x,\theta) = 0$$

- But there is no 3-loop divergences in $D = 4$ [Bern et al., Tourkine, Vanhove]

- It could be the off-shell properties of $\mathcal{N} = 4$ supergravity coupled to 6 vector multiplets make this coupling a protected F-term

Duality symmetries of $\mathcal{N} = 4$ pure supergravity

$\mathcal{N} = 4$ supergravity is special because of the $U(1)$ R-symmetry anomaly [Marcus].

Therefore the $SU(1,1)$ duality symmetry is broken in perturbation and more general functions of the axion-dilaton $S \in SU(1,1)/U(1)$ are allowed for the coefficient of the counter-term to the effective action

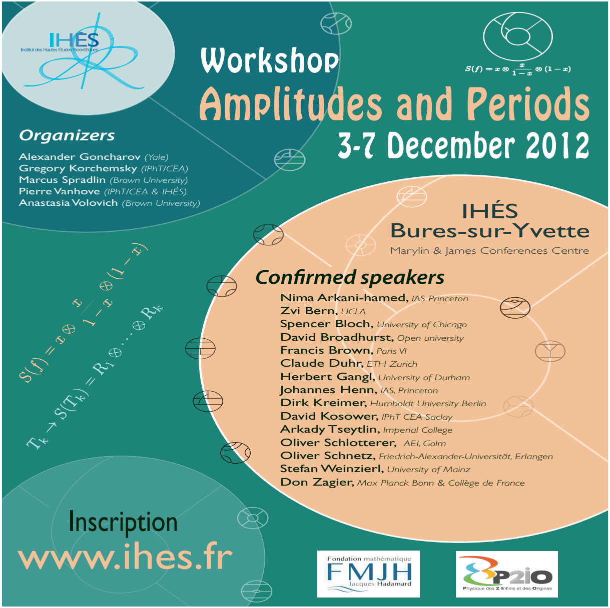
$$\int d^4x \int d^{16}\theta E(x,\theta) h(\mathbb{S}) = \int d^4x \, e \, g(S,\bar{S}) \, \mathcal{R}^4 + \text{susy completion}$$

In string theory $g(S,\bar{S})$ is an modular form invariance under $\Gamma \subset SL(2,\mathbb{Z})$

- The absence of three loop divergences shows that $\Delta_S g(S,\bar{S}) \neq 0$ where $\Delta_S = 4(\Im\mathrm{m} S)^2 \partial_S \bar{\partial}_{\bar{S}}$

Outlook

- Using string theory we have put constraints on the possible counter-terms for UV divergences of four gravitons amplitudes in $\mathcal{N} = 4$ and $\mathcal{N} = 8$ supergravity
- Absence of UV divergences is explained by fermionic zero mode saturation, which is a manifestation of supersymmetry
- In the gravity sector $\mathcal{N} = 4$ amplitudes are the same in the $(4, 0)$ and $(2, 2)$ construction: could be good to be tested from the double copy construction
- At the quantum level $\mathcal{N} = 4$ supergravity is ambiguous because of the anomaly. The ambiguity is seen in the coupling with the scalar fields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